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Nonlinear Rayleigh instability of cylindrical flow with mass and heat transfer

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Abstract
The nonlinear Rayleigh instability of the cylindrical interface between the vapour and liquid phases of a fluid is studied when the phases are enclosed between two cylindrical surfaces coaxial with the interface, and when there is mass and heat transfer across the interface. The method of multiple expansion is used for the investigation. The evolution of amplitude is shown to be governed by a first-order nonlinear differential equation. The stability criterion is discussed, and the region of stability is displayed graphically.

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1. Introduction

This paper investigates the nonlinear stability of the cylindrical interface between the vapour and liquid phases of a fluid when there is mass and heat transfer across the interface.

The problem of stability of liquids when there is mass and heat transfer across the interface has been investigated by several researchers (Hsieh 1972, 1978, 1979, Gill *et al* 1995, Elhefnawy *et al* 1997, Nayak and Chakraborty 1984, Elhefnawy 1994). Hsieh (1978) established a general formulation of the interfacial flow problem with mass and heat transfer and applied it to the Rayleigh–Taylor and Kelvin–Helmholtz instability problems in plane geometry.

In the nuclear reactor cooling of fuel rods by liquid coolants, the geometry of the system in many cases is cylindrical. We have, therefore, considered the interfacial stability problem of a cylindrical flow with heat and mass transfer. Nayak and Chakraborty (1984) studied the Kelvin–Helmholtz stability of the cylindrical interface between the vapour and liquid phases of a fluid, when there is mass and heat transfer across the interface. On the other hand, Elhefnawy (1994) studied the effect of a periodic radial magnetic field on the Kelvin–Helmholtz stability of the cylindrical interface between two magnetic fluids when there is mass and heat transfer across the interface.

The effect of mass and heat transfer across the interface should be taken into account in stability discussions, when the situations are like film boiling of fluids. However, with linear

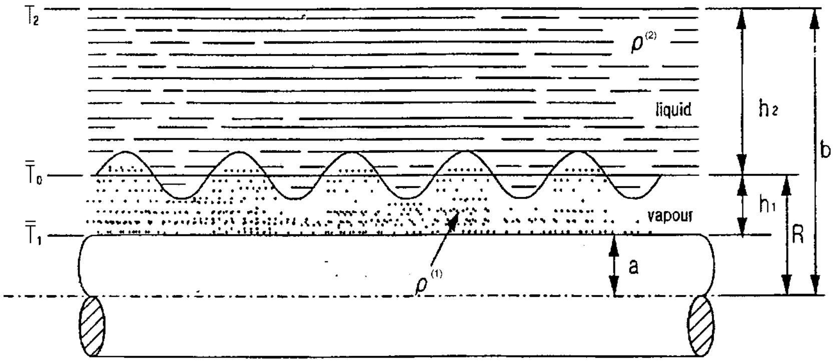

Figure 1. Configuration under consideration in film boiling.

analysis, the stability criterion remains the same as in the case neglecting heat and mass transfer across the interface. Hsieh (1972) found that when the vapour region is hotter than the liquid region, as is usually the case, the effect of mass and heat transfer tends to inhibit the growth of instability. Thus, it is clear that such a uniform model based on linear theory is inadequate to explain the mechanism involved, and nonlinear theory is needed to reveal the effect of heat and mass transfer on the stability of the system. This problem is of fundamental importance in a number of applications such as the design of many types of contacting equipment, e.g., boilers, condensers, reactors and others in industrial and environmental processes.

The basic equations with the accompanying boundary conditions are given in section 2. The first-order theory and the linear dispersion relation are obtained in section 3. In section 4, we have derived second-order solutions. In section 5, a first-order nonlinear differential equation which can be easily integrated is obtained, and in section 6 some numerical examples are presented in graphical forms.

2. Formulation of the problem and basic equations

We shall use a cylindrical system of coordinates (r, θ, z) so that in the equilibrium state the z-axis is the axis of symmetry of the system. The central solid core has a radius a. In the equilibrium state the fluid phase '1', of density $\rho^{(1)}$, occupies the region $a < r < R$, and, the fluid phase '2', of density $\rho^{(2)}$, occupies the region $R < r < b$. The temperature at $r = a$, $r = R$ and $r = b$ is taken as T_1, T_0 and T_2, respectively. The bounding surfaces $r = a$, and $r = b$ are taken as rigid. The interface, after a disturbance, is given by the equation

$$F(r, z, t) = r - R - \eta = 0 \tag{2.1}$$

where η is the perturbation in the radius of the interface from its equilibrium value R, and for which the outward normal vector is written as

$$\mathbf{n} = \frac{\nabla F}{|\nabla F|} = \left\{1 + \left(\frac{\partial \eta}{\partial z}\right)^2\right\}^{-1/2} \left(\mathbf{e}_r - \frac{\partial \eta}{\partial z}\mathbf{e}_z\right). \tag{2.2}$$

We assume that the fluid velocity is irrotational in the region so that velocity potentials are $\phi^{(1)}$ and $\phi^{(2)}$ for fluid phases 1 and 2. In each fluid phase

$$\nabla^2\phi^{(j)} = 0 \qquad (j = 1, 2). \tag{2.3}$$

The solutions for $\phi^{(j)}$ $(j = 1, 2)$ have to satisfy the boundary conditions. The relevant boundary conditions for our configuration are (see figure 1)

(i) On the rigid boundaries $r = a$ and $r = b$:
The normal field velocities vanish on both the central solid core and the outer bounding surface.

$$\frac{\partial \phi^{(1)}}{\partial r} = 0 \qquad \text{on} \quad r = a \tag{2.4}$$

$$\frac{\partial \phi^{(2)}}{\partial r} = 0 \qquad \text{on} \quad r = b. \tag{2.5}$$

(ii) On the interface $r = R + \eta(z, t)$:

(1) The conservation of mass across the interface:

$$\left[\!\left[\rho \left(\frac{\partial F}{\partial t} + \nabla \phi \cdot \nabla F \right) \right]\!\right] = 0 \qquad \text{or} \qquad \left[\!\left[\rho \left(\frac{\partial \phi}{\partial r} - \frac{\partial \eta}{\partial t} - \frac{\partial \eta}{\partial z}\frac{\partial \phi}{\partial z} \right) \right]\!\right] = 0 \tag{2.6}$$

where $[\![h]\!]$ represents the difference in a quantity as we cross the interface, i.e., $[\![h]\!] = h^{(2)} - h^{(1)}$, where the superscripts refer to upper and lower fluids, respectively.

(2) The interfacial condition for energy is

$$L\rho^{(1)} \left(\frac{\partial F}{\partial t} + \nabla \phi^{(1)} \cdot \nabla F \right) = S(\eta) \tag{2.7}$$

where L is the latent heat released when the fluid is transformed from phase 1 to phase 2. Physically, the left-hand side of (2.7) represents the latent heat released during the phase transformation, while $S(\eta)$ on the right-hand side of (2.7) represents the net heat flux, so that the energy will be conserved.

In the equilibrium state, the heat fluxes in the direction of increasing r in the fluid phases 1 and 2 are $-K_1(T_1 - T_0)/R \log(a/R)$ and $-K_2(T_0 - T_2)/R \log(R/b)$, respectively, where K_1 and K_2 are the heat conductivities of the two fluids. As in Hsieh (1978), we denote

$$S(\eta) = \frac{K_2(T_0 - T_2)}{(R + \eta)(\log b - \log(R + \eta))} - \frac{K_1(T_1 - T_0)}{(R + \eta)(\log(R + \eta) - \log a)} \tag{2.8}$$

and we expand it about $r = R$ by Taylor's expansion, such as

$$S(\eta) = S(0) + \eta S'(0) + \tfrac{1}{2}\eta^2 S''(0) + \cdots \tag{2.9}$$

and we take $S(0) = 0$, so that

$$\frac{K_2(T_0 - T_2)}{R \log(b/R)} = \frac{K_1(T_1 - T_0)}{R \log(R/a)} = G \text{ (say)} \tag{2.10}$$

indicating that in the equilibrium state the heat fluxes are equal across the interface in the two fluids.

From (2.1), (2.7) and (2.9), we have

$$\rho^{(1)} \left(\frac{\partial \phi^{(1)}}{\partial r} - \frac{\partial \eta}{\partial t} - \frac{\partial \eta}{\partial z}\frac{\partial \phi^{(1)}}{\partial z} \right) = \alpha(\eta + \alpha_2\eta^2 + \alpha_3\eta^3) \tag{2.11}$$

where

$$\alpha = \frac{G \log(b/a)}{LR \log(b/R) \log(R/a)} \qquad \alpha_2 = \frac{1}{R}\left(-\frac{3}{2} + \frac{1}{\log(b/R)} - \frac{1}{\log(R/a)} \right)$$

$$\alpha_3 = \frac{1}{R^2}\left[\frac{11}{6} - \frac{2\log(R^2/ab)}{\log(b/R)\log(R/a)} + \frac{\log^3(b/R) + \log^3(R/a)}{\{\log(b/R)\log(R/a)\}^2 \log(b/a)} \right].$$

(3) The conservation of momentum balance, by taking into account the mass transfer across the interface, is

$$\rho^{(1)}(\nabla\phi^{(1)}\cdot\nabla F)\left(\frac{\partial F}{\partial t}+\nabla\phi^{(1)}\cdot\nabla F\right)=\rho^{(2)}(\nabla\phi^{(2)}\cdot\nabla F)\left(\frac{\partial F}{\partial t}+\nabla\phi^{(2)}\cdot\nabla F\right)$$
$$+(p_2-p_1+\sigma\nabla\cdot\mathbf{n})|\nabla F|^2 \qquad (2.12)$$

where p is the pressure and σ is the surface tension coefficient, respectively.

By eliminating the pressure using Bernoulli's equation, we can rewrite the above condition (2.12) as

$$\left[\!\left[\rho\left\{\frac{\partial\phi}{\partial t}+\frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2+\frac{1}{2}\left(\frac{\partial\phi}{\partial z}\right)^2-\left\{1+\left(\frac{\partial\eta}{\partial z}\right)^2\right\}^{-1}\right.\right.\right.$$
$$\left.\left.\left.\times\left(\frac{\partial\phi}{\partial z}\frac{\partial\eta}{\partial z}-\frac{\partial\phi}{\partial r}\right)\left(\frac{\partial\eta}{\partial t}+\frac{\partial\phi}{\partial z}\frac{\partial\eta}{\partial z}-\frac{\partial\phi}{\partial r}\right)\right\}\right]\!\right]$$
$$=-\sigma\frac{\partial^2\eta}{\partial z^2}\left\{1+\left(\frac{\partial\eta}{\partial z}\right)^2\right\}^{-3/2}+\sigma(R+\eta)^{-1}\left\{1+\left(\frac{\partial\eta}{\partial z}\right)^2\right\}^{-1/2}. \qquad (2.13)$$

To investigate the nonlinear effects on the stability of the system, we employ the method of multiple scales (Lee 1997, 1999a, 1999b). Introducing ϵ as a small parameter, we assume the following expansion of the variables:

$$\eta=\sum_{n=1}^{3}\epsilon^n\eta_n(z,t_0,t,t_2)+O(\epsilon^4) \qquad (2.14)$$

$$\phi^{(j)}=\sum_{n=1}^{3}\epsilon^n\phi_n^{(j)}(r,z,t_0,t_1,t_2)+O(\epsilon^4) \qquad (j=1,2) \qquad (2.15)$$

where $t_n=\epsilon^n t$ $(n=0,1,2)$. The quantities appearing in the field equations (2.3) and the boundary conditions (2.6), (2.11) and (2.13) can now be expressed in Maclaurin series expansion around $r=R$. Then, we use (2.14) and (2.15) and equate the coefficients of equal power series in ϵ to obtain the linear and the successive nonlinear partial differential equations of various orders (see the appendix).

3. Linear theory

The linear wave solutions of (2.3) subject to boundary conditions yield

$$\eta_1=A(t_1,t_2)\,\mathrm{e}^{\mathrm{i}\theta}+\bar{A}(t_1,t_2)\,\mathrm{e}^{-\mathrm{i}\theta} \qquad (3.1)$$

$$\phi_1^{(1)}=\frac{1}{k}\left(\frac{\alpha}{\rho^{(1)}}-\mathrm{i}\omega\right)A(t_1,t_2)E^{(1)}(k,r)\,\mathrm{e}^{\mathrm{i}\theta}+\mathrm{c.c.} \qquad (3.2)$$

$$\phi_1^{(2)}=\frac{1}{k}\left(\frac{\alpha}{\rho^{(2)}}-\mathrm{i}\omega\right)A(t_1,t_2)E^{(2)}(k,r)\,\mathrm{e}^{\mathrm{i}\theta}+\mathrm{c.c.} \qquad (3.3)$$

where

$$E^{(1)}(k,r)=\frac{I_0(kr)K_1(ka)+I_1(ka)K_0(kr)}{I_1(kR)K_1(ka)-I_1(ka)K_1(kR)} \qquad (3.4)$$

$$E^{(2)}(k,r)=\frac{I_0(kr)K_1(kb)+I_1(kb)K_0(kr)}{I_1(kR)K_1(kb)-I_1(kb)K_1(kR)}$$
$$\theta=kz-\omega t_0 \tag{3.5}$$

with I_m and K_m ($m = 0, 1$) being the modified Bessel functions of the first and second kinds, respectively.

Substituting (3.1)–(3.3) into (2.13), we obtain the following dispersion relation:

$$D(\omega,k)=a_0\omega^2+\mathrm{i}a_1\omega-a_2=0 \tag{3.6}$$

where

$$a_0=\rho^{(1)}E^{(1)}(k,R)-\rho^{(2)}E^{(2)}(k,R) \qquad a_1=\alpha\{E^{(1)}(k,R)-E^{(2)}(k,R)\}$$
$$a_2=\frac{\sigma k}{R^2}(R^2k^2-1).$$

From the properties of Bessel functions, and since α is always positive, we note that $a_0 > 0$ and $a_1 > 0$. The condition for stability is $a_2 > 0$, from which we obtain $k > \frac{1}{R}$. Thus the system is stable if $k > k_c$, where

$$k_c=\frac{1}{R}. \tag{3.7}$$

4. Second-order solutions

With the use of the first-order solutions, we obtained the equations for the second-order problem

$$\nabla^2\phi_2^{(j)}=0 \qquad (j=1,2) \tag{4.1}$$

and the boundary conditions at $r = R$:

$$\rho^{(j)}\left\{\frac{\partial\phi_2^{(j)}}{\partial r}-\frac{\partial\eta_2}{\partial t_0}\right\}-\alpha\eta_2=\left[\rho^{(j)}\left\{\frac{\alpha}{\rho^{(j)}}-\mathrm{i}\omega\right\}\left\{\frac{1}{R}-2kE^{(j)}(k,R)\right\}+\alpha\alpha_2\right]A^2\,\mathrm{e}^{2\mathrm{i}\theta}$$
$$+\rho^{(j)}\frac{\partial A}{\partial t_1}\,\mathrm{e}^{\mathrm{i}\theta}+\text{c.c.}+2\alpha\left(\frac{1}{R}+\alpha_2\right)|A|^2 \qquad (j=1,2) \tag{4.2}$$

$$\rho^{(2)}\frac{\partial\phi_2^{(2)}}{\partial r}-\rho^{(1)}\frac{\partial\phi_2^{(1)}}{\partial r}-\{\rho^{(2)}-\rho^{(1)}\}\frac{\partial\eta_2}{\partial t_0}=\left[\!\left[\rho\left(\frac{\alpha}{\rho}-\mathrm{i}\omega\right)\left\{\frac{1}{R}-2kE(k,R)\right\}\right]\!\right]A^2\,\mathrm{e}^{2\mathrm{i}\theta}$$
$$+\{\rho^{(2)}-\rho^{(1)}\}\frac{\partial A}{\partial t_1}\,\mathrm{e}^{\mathrm{i}\theta}+\text{c.c.} \tag{4.3}$$

$$\rho^{(2)}\frac{\partial\phi_2^{(2)}}{\partial t_0}-\rho^{(1)}\frac{\partial\phi_2^{(1)}}{\partial t_0}+\sigma\left(\frac{\partial^2\eta_2}{\partial z_0^2}+\frac{\eta_2}{R^2}\right)=\left\{-\frac{\omega^2}{2}[\![\rho\{E^2(k,R)-3\}]\!]\right.$$
$$\left.+\frac{\alpha^2}{2}\left[\!\left[\frac{1+E^2(k,R)}{\rho}\right]\!\right]-\mathrm{i}\alpha\omega[\![E^2(k,R)]\!]+\frac{\sigma}{2R^3}(R^2k^2+2)\right\}A^2\,\mathrm{e}^{2\mathrm{i}\theta}$$
$$-\left[\!\left[\frac{\rho}{k}\left(\frac{\alpha}{\rho}-\mathrm{i}\omega\right)E(k,R)\right]\!\right]\frac{\partial A}{\partial t_1}\,\mathrm{e}^{\mathrm{i}\theta}+\text{c.c.}$$
$$+\left\{\left[\!\left[\rho\left(\frac{\alpha^2}{\rho^2}+\omega^2\right)\{1-E^2(k,R)\}\right]\!\right]-\frac{\sigma}{R^3}(R^2k^2-2)\right\}|A|^2. \tag{4.4}$$

The non-secularity condition for the existence of the uniformly valid solution is

$$\frac{\partial A}{\partial t_1}=0. \tag{4.5}$$

Equations (4.1)–(4.4) furnish the second-order solutions:

$$\eta_2 = -2\left(\frac{1}{R} + \alpha_2\right)|A|^2 + A_2 A^2 \,\mathrm{e}^{2\mathrm{i}\theta} + \bar{A}_2 \bar{A}^2 \,\mathrm{e}^{-2\mathrm{i}\theta} \tag{4.6}$$

$$\phi_2^{(j)} = B_2^{(j)} A^2 \,\mathrm{e}^{2\mathrm{i}\theta} E^{(j)}(2k, r) + \text{c.c.} + b^{(j)}(t_0, t_1, t_2) \qquad (j = 1, 2) \tag{4.7}$$

where

$$A_2 = \frac{2k}{D(2\omega, 2k)} \left\{ \left[\!\!\left[\rho \frac{\mathrm{i}\omega}{k} E(2k, R)\beta + \frac{\rho}{2} E^2(k, R) \left(\frac{\alpha}{\rho} - \mathrm{i}\omega\right)^2 + \frac{3\omega^2\rho^2 + \alpha^2}{2\rho} \right]\!\!\right] + \frac{\sigma}{2R^3}(2 + R^2k^2) \right\} \tag{4.8}$$

$$B_2^{(j)} = \frac{1}{2k}\left[\beta^{(j)} + \left\{\frac{\alpha}{\rho^{(j)}} - 2\omega\mathrm{i}\right\} A_2\right] \tag{4.9}$$

$$\beta^{(j)} = k\left\{\frac{\alpha}{\rho^{(j)}} - \mathrm{i}\omega\right\}\left\{\frac{1}{kR} - 2E^{(j)}(k, R)\right\} + \frac{\alpha\alpha_2}{\rho^{(j)}} \tag{4.10}$$

$$\rho^{(2)}\frac{\partial b^{(2)}}{\partial t_0} - \rho^{(1)}\frac{\partial b^{(1)}}{\partial t_0} = \left\{ \left[\!\!\left[\rho\left(\frac{\alpha^2}{\rho^2} + \omega^2\right)\{1 - E^2(k, R)\} \right]\!\!\right] - \frac{\sigma}{R^3}(k^2R^2 - 4 - 2R\alpha_2) \right\} |A|^2 \tag{4.11}$$

and we have assumed that $D(2\omega, 2k) \neq 0$. The case when $D(2\omega, 2k) = 0$ corresponds to the second harmonic resonance.

5. Third-order solutions

We examine now the third-order problem:

$$\nabla_0^2 \phi_3^{(i)} = 0 \qquad (i = 1, 2). \tag{5.1}$$

On substituting the values of $\eta_1, \phi_1^{(i)}$ from (3.1)–(3.3) and $\eta_2, \phi_2^{(i)}$ from (4.6)–(4.7) into (A.8), we obtain

$$\phi_3^{(j)} = C_3^{(j)} E^{(j)}(k, r) A^2 \bar{A} \,\mathrm{e}^{\mathrm{i}\theta} + \frac{E^{(j)}(k, r)}{k} \frac{\partial A}{\partial t_2} \mathrm{e}^{\mathrm{i}\theta} + \text{c.c.} \qquad (j = 1, 2) \tag{5.2}$$

where

$$\begin{aligned} C_3^{(j)} = -k\Bigg[& 2\left\{E^{(j)}(2k, R) - \frac{1}{kR}\right\} B_2^{(j)} - 2\left(E^{(j)}(k, R) - \frac{1}{kR}\right)\left(\frac{\alpha}{\rho^{(j)}} - \mathrm{i}\omega\right) \\ & \times \left(\frac{1}{kR} + \frac{\alpha_2}{k}\right) + \frac{1}{2}\left(1 + \frac{2}{R^2k^2} - \frac{E^{(j)}(k, R)}{Rk}\right)\left(\frac{3\alpha}{\rho^{(j)}} - \mathrm{i}\omega\right) \\ & - \frac{\alpha}{\rho^{(j)}} - \mathrm{i}\omega + \frac{\alpha}{\rho^{(j)}k^2}\left\{4\alpha_2\left(\frac{1}{R} + \alpha_2\right) - 3\alpha_3\right\} \\ & - \left\{\left(\frac{\alpha}{\rho^{(j)}} + \mathrm{i}\omega\right)\left(E^{(j)}(k, R) + \frac{1}{Rk}\right) + \frac{2\alpha\alpha_2}{\rho^{(j)}k}\right\}\frac{A_2}{k}\Bigg]. \end{aligned} \tag{5.3}$$

We substitute the first- and second-order solutions into the third-order equation. In order to avoid non-uniformity of the expansion, we again impose the condition that secular terms vanish. Then from (A.9), we find

$$\frac{\mathrm{i}}{k}\frac{\partial D}{\partial \omega}\frac{\partial A}{\partial t_2} + qA^2\bar{A} = 0 \tag{5.4}$$

where

$$\begin{aligned} q = {} & \left[\!\!\left[\rho \left\{ -\mathrm{i}\omega C_3 E(k, R) - A_2 \mathrm{i}\omega \left(\frac{\alpha}{\rho} + \mathrm{i}\omega\right) + 2kB_2 \left(\frac{\alpha}{\rho}\{E(k, R)E(2k, R) - 1\}\right.\right.\right.\right. \\ & \left. + \mathrm{i}\omega\{E(k, R)E(2k, R) - 2\}\right) + 2\,\mathrm{i}\omega\left(\frac{1}{R} + \alpha_2\right)\left(\frac{\alpha}{\rho} - \mathrm{i}\omega\right) \\ & \left.\left. + kE(k, R)\frac{\mathrm{i}\omega}{2}\left(\frac{3\alpha}{\rho} - \mathrm{i}\omega\right) + \frac{1}{2R}\left(3\omega^2 + \frac{6\alpha^2}{\rho^2} - \mathrm{i}\frac{\alpha}{\rho}\omega\right)\right\}\right]\!\!\right] \\ & + \frac{\sigma}{R^4}\left\{2RA_2(k^2R^2 - 1) + 4R\alpha_2 + 7 - \frac{1}{2}k^2R^2(1 - 3k^2R^2)\right\}. \end{aligned} \tag{5.5}$$

We rewrite (5.4) as

$$\frac{\partial A}{\partial t_2} + QA^2\bar{A} = 0 \tag{5.6}$$

which can be easily integrated as

$$|A|^2 = \frac{1}{|A_0|^{-2} + 2(\mathrm{Re}\, Q)t_2} \tag{5.7}$$

where A_0 is the initial amplitude and Re Q means the real part of Q. With a finite initial value A_0, A may become infinite when the denominator in (5.7) vanishes. Otherwise, A will be bounded. If Re Q is negative, the denominator of (5.7) can vanish and it implies rupture. Thus, the stability condition is

$$\mathrm{Re}\, Q > 0. \tag{5.8}$$

6. Numerical results

The stability of the system depends on condition (5.8). We show the stable and unstable regions in figures 2–5. In figures 2 and 3, the dotted lines represent the linear curve which divides the plane into a stable region (above the curve), and an unstable region (below the curve).

The shaded regions S are the nonlinearly stabilized regions, while the regions indicated by U represent the destabilized regions which were originally stable regions in the linear theory.

Figures 2–5 show the stability diagrams in the h_1–k plane corresponding to the cases $\alpha = 6 \times 10^{-2}, 10^{-1}, 6 \times 10^{-1},\ 1$ g cm^{-3} s^{-1}, $\sigma = 0.06$ dyn cm^{-1}, $a = 1$ cm and $b = 2$ cm. These figures are similar in shape. Unlike the linear theory, the stability regions form narrow bands which shift upwards, and the breadth of the bands broadens as α increases, and thus in the nonlinear theory the region of stability is much reduced.

Therefore, from these figures we note that as α increases it has a stabilizing effect, and the mass and heat transfer has a destabilizing effect for larger values of k.

From the above numerical discussion, it is evident that, apart from the effect of the variation of h_1 and h_2, the mass and heat transfer coefficient α plays an important role in stability criteria in contrast to linear theory. In the linear theory the mass and heat transfer coefficient α has no effect on the linear stability conditions.

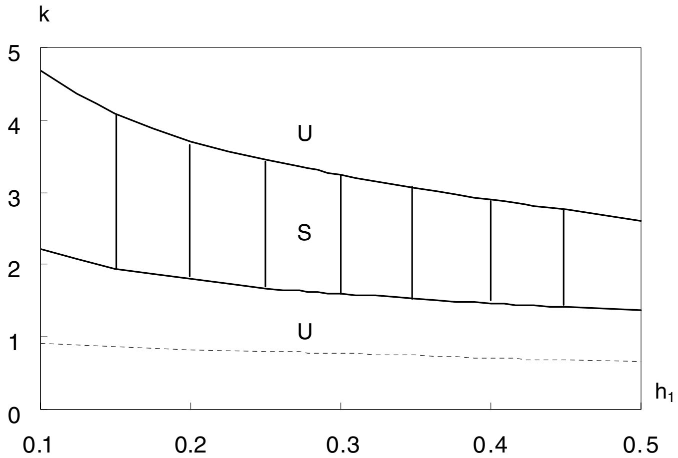

Figure 2. Stability diagram in the h_1–k plane for a system having $\rho_1 = 3.652 \times 10^{-4}$ g cm^{-3}, $\rho_2 = 5.97 \times 10^{-2}$ g cm^{-3}, $\alpha = 6 \times 10^{-2}$ g cm^{-3} s^{-1}.

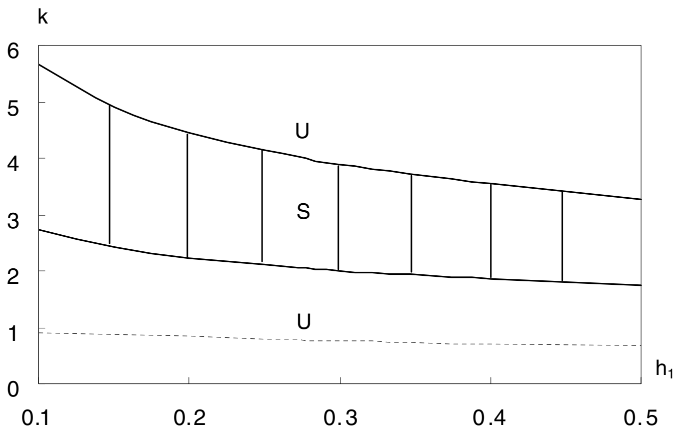

Figure 3. Stability diagram in the h_1–k plane for a system having $\rho_1 = 3.652 \times 10^{-4}$ g cm^{-3}, $\rho_2 = 5.97 \times 10^{-2}$ g cm^{-3}, $\alpha = 1 \times 10^{-1}$ g cm^{-3} s^{-1}.

The present method has an advantage over the method originally used by Hsieh (1979) by which he investigated the nonlinear solution near the cut-off wave number. His method is simpler than the present method; however it has a definite disadvantage in that the information on the behaviour of the flow can be obtained only at the cut-off wave numbers, whereas the present method is improved and free of such a shortcoming. Such a shortcoming also exists in the solution of the Ginzburg–Landau equation obtained by Elhefnawy *et al* (1997).

In figures 2–5, the thickness of the vapour varies from 0.1 cm to 0.5 cm. From figure 2, we can see that the region of stability is narrower than the case of figure 5 where α is much larger. This implies that when the heat flux is sufficiently intense, the system can be stabilized. Moreover, from these figures we can see that the region of stability reduces as the thickness of the vapour increases. So, with the same heat flux, the thinner the vapour layer, the more easily the system can be stabilized. To sum up, we have presented an analysis of nonlinear Rayleigh instability. It is found that, unlike in linear theory, the stability region exists in the

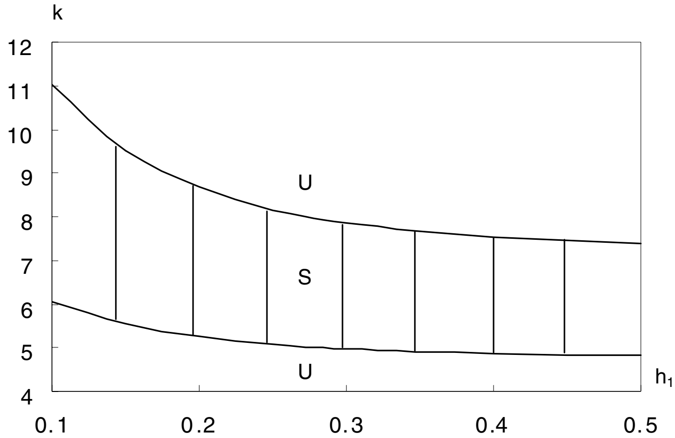

Figure 4. Stability diagram in the h_1–k plane for a system having $\rho_1 = 3.652 \times 10^{-4}$ g cm^{-3}, $\rho_2 = 5.97 \times 10^{-2}$ g cm^{-3}, $\alpha = 6 \times 10^{-1}$ g cm^{-3} s^{-1}.

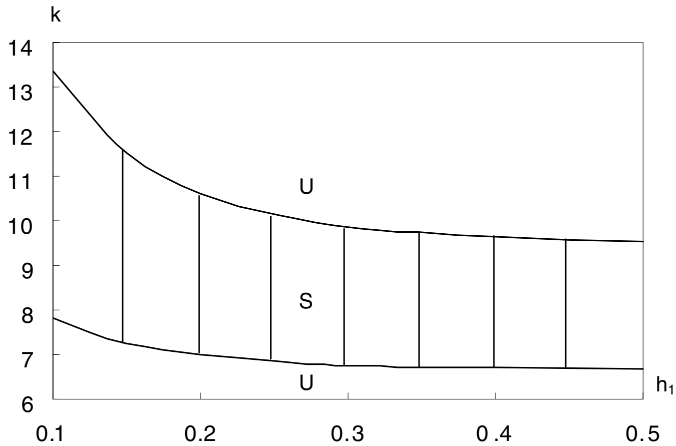

Figure 5. Stability diagram in the h_1–k plane for a system having $\rho_1 = 3.652 \times 10^{-4}$ g cm^{-3}, $\rho_2 = 5.97 \times 10^{-2}$ g cm^{-3}, $\alpha = 1.0$ g cm^{-3} s^{-1}.

form of a narrow band, the width of which reduces with the increment of the thickness of the vapour layer. The region of stability is enlarged with stronger heat flux.

7. Conclusions

The stability of liquids in a cylindrical flow when there is mass and heat transfer across the interface which depicts film boiling is studied. Using the method of multiple time scales, a first-order nonlinear differential equation describing the evolution of nonlinear waves is obtained. Unlike linear theory, with nonlinear theory, it is evident that the mass and heat transfer plays an important role in the stability of fluids, in a situation such as film boiling.

Acknowledgment

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Appendix

The interfacial conditions are given on $r = R$ as
Order $O(\epsilon)$

$$\left[\!\left[\rho \left(\frac{\partial \phi_1}{\partial r} - \frac{\partial \eta_1}{\partial t_0} \right) \right]\!\right] = 0 \tag{A.1}$$

$$\rho^{(1)} \left(\frac{\partial \phi_1^{(1)}}{\partial r} - \frac{\partial \eta_1}{\partial t_0} \right) = \alpha \eta_1 \tag{A.2}$$

$$\left[\!\left[\rho \frac{\partial \phi_1}{\partial t_0} \right]\!\right] = -\sigma \left(\frac{\partial^2 \eta_1}{\partial z^2} + \frac{\eta_1}{R^2} \right). \tag{A.3}$$

Order $O(\epsilon^2)$

$$\left[\!\left[\rho \left(\frac{\partial \phi_2}{\partial r} + \frac{\partial^2 \phi_1}{\partial r^2}\eta_1 - \frac{\partial \eta_2}{\partial t_0} - \frac{\partial \eta_1}{\partial t_1} - \frac{\partial \eta_1}{\partial z}\frac{\partial \phi_1}{\partial z} \right) \right]\!\right] = 0 \tag{A.4}$$

$$\rho^{(1)} \left(\frac{\partial \phi_2^{(1)}}{\partial r} + \frac{\partial^2 \phi_1^{(1)}}{\partial r^2}\eta_1 - \frac{\partial \eta_2}{\partial t_0} - \frac{\partial \eta_1}{\partial t_1} - \frac{\partial \eta_1}{\partial z}\frac{\partial \phi_1^{(1)}}{\partial z} \right) = \alpha \left(\eta_2 + \alpha_2 \eta_1^2 \right) \tag{A.5}$$

$$\begin{aligned} &\left[\!\left[\rho \left\{ \frac{\partial \phi_2}{\partial t_0} + \frac{\partial \phi_1}{\partial t_1} + \frac{\partial^2 \phi_1}{\partial t_0 \partial r}\eta_1 + \frac{1}{2}\left[\left(\frac{\partial \phi_1}{\partial r} \right)^2 + \left(\frac{\partial \phi_1}{\partial z} \right)^2 \right] + \frac{\partial \phi_1}{\partial r}\left(\frac{\partial \eta_1}{\partial t_0} - \frac{\partial \phi_1}{\partial r} \right) \right\} \right]\!\right] \\ &\quad = -\sigma \left\{ \frac{\partial^2 \eta_2}{\partial z^2} + \frac{1}{2R}\left(\frac{\partial \eta_1}{\partial z} \right)^2 + \frac{\eta_2}{R^2} - \frac{\eta_1^2}{R^3} \right\}. \end{aligned} \tag{A.6}$$

Order $O(\epsilon^3)$

$$\begin{aligned} &\left[\!\left[\rho \left\{ \frac{\partial \phi_3}{\partial r} + \frac{\partial^2 \phi_2}{\partial r^2}\eta_1 + \frac{\partial^2 \phi_1}{\partial r^2}\eta_2 + \frac{1}{2}\frac{\partial^3 \phi_1}{\partial r^3}\eta_1^2 - \frac{\partial \eta_3}{\partial t_0} - \frac{\partial \eta_2}{\partial t_1} - \frac{\partial \eta_1}{\partial t_2} \right. \right.\right. \\ &\quad \left.\left.\left. - \frac{\partial \eta_1}{\partial z}\left(\frac{\partial \phi_2}{\partial z} + \frac{\partial^2 \phi_1}{\partial z \partial r}\eta_1 \right) - \frac{\partial \eta_2}{\partial z}\frac{\partial \phi_1}{\partial z} \right\} \right]\!\right] = 0 \end{aligned} \tag{A.7}$$

$$\begin{aligned} &\rho^{(1)} \left\{ \frac{\partial \phi_3^{(1)}}{\partial r} + \frac{\partial^2 \phi_2^{(1)}}{\partial r^2}\eta_1 + \frac{\partial^2 \phi_1^{(1)}}{\partial r^2}\eta_2 + \frac{1}{2}\frac{\partial^3 \phi_1^{(1)}}{\partial r^3}\eta_1^2 - \frac{\partial \eta_3}{\partial t_0} - \frac{\partial \eta_2}{\partial t_1} - \frac{\partial \eta_1}{\partial t_2} \right. \\ &\quad \left. - \frac{\partial \eta_1}{\partial z}\left(\frac{\partial \phi_2^{(1)}}{\partial z} + \frac{\partial^2 \phi_1^{(1)}}{\partial z \partial r}\eta_1 \right) - \frac{\partial \eta_2}{\partial z}\frac{\partial \phi_1^{(1)}}{\partial z} \right\} = \alpha \left(\eta_3 + 2\alpha_2 \eta_1 \eta_2 + \alpha_3 \eta_1^3 \right) \end{aligned} \tag{A.8}$$

$$\begin{aligned} &\left[\!\left[\rho \left\{ \frac{\partial \phi_3}{\partial t_0} + \frac{\partial \phi_2}{\partial t_1} + \frac{\partial \phi_1}{\partial t_2} + \frac{\partial^2 \phi_1}{\partial t_0 \partial r}\eta_2 + \left(\frac{\partial^2 \phi_1}{\partial t_1 \partial r} + \frac{\partial^2 \phi_2}{\partial t_0 \partial r} \right)\eta_1 + \frac{1}{2}\frac{\partial^3 \phi_1}{\partial t_0 \partial r^2}\eta_1^2 \right.\right.\right. \\ &\quad + \frac{\partial \phi_1}{\partial r}\left(\frac{\partial \phi_2}{\partial r} + \frac{\partial^2 \phi_1}{\partial r^2}\eta_1 \right) + \frac{\partial \phi_1}{\partial z}\left(\frac{\partial \phi_2}{\partial z} + \frac{\partial^2 \phi_1}{\partial r \partial z}\eta_1 \right) \end{aligned}$$

$$-\left(\frac{\partial\phi_1}{\partial r}-\frac{\partial\eta_1}{\partial t_0}\right)\left(\frac{\partial\phi_2}{\partial r}-\frac{\partial\phi_1}{\partial z}\frac{\partial\eta_1}{\partial z}\right)+\frac{\partial\phi_1}{\partial r}\left(\frac{\partial\eta_2}{\partial t_0}+\frac{\partial\eta_1}{\partial t_1}-\frac{\partial\phi_2}{\partial r}+\frac{\partial\phi_1}{\partial z}\frac{\partial\eta_1}{\partial z}\right)$$

$$+\eta_1\frac{\partial^2\phi_1}{\partial r^2}\left(\frac{\partial\eta_1}{\partial t_0}-2\frac{\partial\phi_1}{\partial r}\right)\Bigg\}\Bigg]\!\!\Bigg] = -\sigma\left\{\frac{\partial^2\eta_3}{\partial z^2}-\frac{3}{2}\frac{\partial^2\eta_1}{\partial z^2}\left(\frac{\partial\eta_1}{\partial z}\right)^2\right.$$

$$\left.-\frac{1}{2}\frac{\eta_1}{R^2}\left(\frac{\partial\eta_1}{\partial z}\right)^2+\frac{1}{R}\frac{\partial\eta_1}{\partial z}\frac{\partial\eta_2}{\partial z}+\frac{\eta_3}{R^2}-\frac{2\eta_1\eta_2}{R^3}+\frac{\eta_1^3}{R^4}\right\}. \tag{A.9}$$

References

Elhefnawy A R F 1994 Stability properties of a cylindrical flow in magnetic fluids: effect of mass and heat transfer and periodic radial field *Int. J. Eng. Sci.* **32** 805–15

Elhefnawy A R F, El-Dib Y O and Mahmoud Y S 1997 Nonlinear electrohydrodynamic Rayleigh–Taylor instability with mass and heat transfer subject to a vertical oscillating force and a horizontal electric field *Int. J. Theor. Phys.* **36** 2079–97

Gill Gurpreet K, Chhabra R K and Trehan S K 1995 Bubble formation in superposted magnetic fluids in the presence of heat and mass transfer *Z. Naturf.* a **50** 805–12

Hsieh D Y 1972 Effect of heat and mass transfer on Rayleigh–Taylor instability *Trans. ASME* D **94** 156–62

Hsieh D Y 1978 Interfacial stability with mass and heat transfer *Phys. Fluids* **21** 745–8

Hsieh D Y 1979 Nonlinear Rayleigh–Taylor stability with mass and heat transfer *Phys. Fluids* **22** 1435–39

Lee D-S 1997 Nonlinear interaction of capillary-gravity waves with a subsonic gas in the presence of magnetic field *J. Physique* II **7** 1045–55

Lee D-S 1999a Nonlinear capillary waves on the surface of an electrohydrodynamic planar jet *Z. Angew. (Math.) Mech.* **79** 627–34

Lee D-S 1999b Second harmonic resonance on the surface of a magnetohydrodynamic fluid column *Z. Naturf.* a **54** 335–42

Lee D-S 2000 Nonlinear stability of a cylindrical interface with mass and heat transfer *Z. Naturf.* a **55** 837–42

Nayak A R and Chakraborty B B 1984 Kelvin–Helmholtz stability with mass and heat transfer *Phys. Fluids* **27** 1937–41